The Rockdale Mathematics Competition Problem Book, 2004-2014

Chuck Garner, Editor

THE ROCKDALE MATHEMATICS COMPETITION PROBLEM BOOK: 2004-2014

First edition, 2015

Published by RMMT Publishing, Conyers, Georgia USA

This book was produced directly from the author's LaTeX files.
Figures were drawn by the author using the TeXdraw package and PSTricks.

Typeset in *TeX Gyre Termes* (a TeX implementation of *Nimbus Roman No. 9*)
and Epigrafica (a TeX implementation of Hermann Zapf's Optima).

Cover photograph: "Rotterdam Cube Houses" by Lode Van de Velde,
courtesy of `www.publicdomainpictures.net`.

Special Thanks to all members of the Rockdale Magnet Math Team.

© Dr. Charles R. Garner, Jr., 2010, 2015

Preface to the New Edition

This book includes all competition papers from the first ten Rockdale Mathematics Competitions—those over the years 2004 to 2014. Some of these competition papers have appeared in an earlier edition of this book, but the new edition brought with it the opportunity to make corrections of some typos in those competition papers, as well as make some changes and additions.

This new edition is printed on standard $8\frac{1}{2} \times 11$ page size to make it easier on teachers to copy materials for practice with their teams. The former 6×9 size was a nice compact little volume, but it was difficult to make copies from those pages. This new edition features five Ciphering problems to a page and the Individual questions separated with horizontal lines. This is so teachers can make copies and then cut the copies so students can practice one set of problems at a time, simulating actual competition conditions. The Relay problems are similarly formatted. The Power and Team Round problems are each on a single page, again to help teachers make copies for their students.

There are other new additions to this volume. Included at the begining of each year's problems is a list of the top 5 team and individual winners of that year's tournament, and the average number of correct responses and scores. There is a new index of topics, terms, and names used in the problems. Finally, there is a special Appendix which lists only the answers, so teachers may use it to quickly check students' answers, or so students can check their own work before reading the solutions.

The format of the tournament has remained ARML-style, with some tweaks. Beginning in 2011, we increased the number of individual and team questions from 8 to 10. This meant the point values and the timing of the rounds changed. The Power Round — to be completed by teams of six students in 40 minutes — and the Team Round — to be completed by a team in 20 minutes — are now each worth 50 points. Each of the two Relay Rounds remained 20 points but the timing changed to 4 minutes per Relay. The Individual Round is now worth 60 points with each pair of problems to be completed within 6 minutes. The maximum point total for a team is therefore 200.

The Individual Round is the sole determining factor in the individual awards. Of course, it is possible for students to get the same number of problems correct; indeed, this always happens. To distinguish among the top 12 award winners, it would seem that we would need to hold a large tie-breaker round. However, the possiblity of such a large round eliminated by weighting the individual problems. Beginning in 2007, the problems were designed to be in order of difficulty, so the easiest problems was #1 and the most difficult was #10. This resulted in many automatic tie-breakers. For instance, say Pat and Chris each got 8 problems correct. If Pat correctly answered 1, 2, 3, 4, 5, 6, 7, and 8, but Chris correctly answered 1, 2, 3, 4, 5, 6, 8, and 9, then Chris would "win" the tie since Chris answered more of the difficult problems correctly.

However, we realized over the years that we didn't always get the order of difficulty quite right! So beginning in 2013, we calculated the difficulty based on how many participants answered each problem incorrectly. This means that if 80% answer #10 incorrectly, but 95% answer problem #9 incorrectly, then we consider #9 more difficult than #10, and break any ties accordingly. (The only tie-breakers left for us to break in a special tie-breaker round are those participants who answer the exact same problems correctly, and we only do this for those under consideration for an award.)

The difficulty of the tournament has varied over the years. At the beginning of each chapter is a short introduction, describing the tournament and the results. Included in this is a summary of the average score on each round. If we use this average score as a measure of difficulty, then some interesting statistics emerge. Below is a table listing the rounds and years of the lowest and highest average scores for the 8 years of the Team, Relay, and Power Rounds, as well as the Total Team Score. (To make the comparisons easier and because the rounds have changed point-values over the years, the average score was converted to a percentage of the points possible on that round.) The Individual Round difficulty was determined by the average percentage of correct responses from all participants. The actual average scores can be found in the introductions to each competition.

Competition Years with the Highest and Lowest Average Scores

	Individual	Team	Relay	Power	Total Team Score
Lowest Middle School Averages	2011	2010	2009	2012	2010
Highest Middle School Averages	2008	2013	2014	2008	2008
Lowest JV Averages	2014	2008	2007	2008	2007
Highest JV Average	2012	2012	2014	2011	2011
Lowest Varsity Averages	2013	2013	2013	2014	2013
Highest Varsity Averages	2014	2009	2008	2011	2009

The authors of the more recent problem sets in this book are the students listed below. They were all students at Rockdale Magnet School when writing these problems. Even though some of the problems they submitted were heavily edited and rewritten by me, their ideas and problem-solving strategies remained. They were wonderful young people who were enthused about running mathematics competitions.

2010 Samantha Allen, Julian Hinds, Jonathan Johnson, Vaidehi Joshi, Faustine Li, Kevin Lin, Valeska Lobo, Aaron Roberts, Ken Sinmura, Edwin White, Ksenia Zakirova

2011 Ksenia Zakirova

2012 Gedeion Addisu, Carmen Candal, Johnathan Davis, Cameron Easley, Ray Garner, Shoman Kasbekar, Gabe Kustick, Jason Mitchell, Jeremy Rachels, Anika Reza, Brian Silva, Anna Grace Slifko, Zachary York, Michelle Zhang

2013 Heeyoon Kim

2014 Jeff Bang, Sidney Billingsley, Jaire Duncan, Ray Garner, Ashley Hong, Carlan Ivey, Bethany Mays, Ashley Robbins, Naomi Siddiquee

Thanks to Michelle Zhang who supplied many of the solutions (in LaTeX) to the 2004 competition. The photographs reproduced in this book were taken by Emily Pace, except for the photo on page 323 which was taken by Jeff Bang.

I am pleased that the Rockdale Mathematics Competition has remained a popular math tournament for students in Georgia. It is a large undertaking. Although I handle the registration and communicate with sponsors, the students do all the rest. In addition to the problems writers listed above, there have been hundreds of students who help run the tournament, functioning as scorers, runners, graders, and food prep workers. Thanks also to the alumni who come back to be proctors. In fact, special recognition should go to Jonathan Johnson. Jonathan attended the 2004 and 2006 tournaments as a middle school student. He then went on to help run the 2007, 2008, and 2009 tournaments during high school. He has also returned to be the emcee at the 2010, 2011, 2013, and 2014 tournaments while attending Georgia Tech. (Yes, he missed the 2012 tournament, but the emcee role was superbly and graciously filled by former team captain Amy Lanchester.) Jonathan holds the student/alumni record for being present at the most RMCs.

Thanks also to everyone who has been a member of the Rockdale Magnet Math Team or a member of the Rockdale County High School chapter of Mu Alpha Theta.

Chuck Garner
CONYERS, GA
JANUARY 2015

Preface to the Original Edition

Contained in this book are all the problems given at the first five Rockdale Mathematics Competitions. The problems contained within were all written and solved by the students of the Rockdale Magnet School's award-winning Math Team. The Competition itself is run and hosted by the Math Team as well. The Competition has evolved over its short life, and so a little history is warranted.

As soon as we had the idea to host our own event, we also realized that we wanted to make this more than a high school competition. We wanted a format that would encourage middle schools to take part. So we decided on a format made popular in Alabama, that of subject area tests: pre-algebra, first-year algebra, geometry, second-year algebra, and a comprehensive division.

The competitive rounds in each division involved a written test and ciphering. The test format was simply a 30-question multiple-choice test with a 70-minute time limit. Students received four points per correct answer, zero points per problem left blank, and -1 points per incorrect answer. The ciphering round format was as follows. One student from each school was selected to answer a total of five problems, each with a two-minute time limit. After the first student, a second students was selected to represent his team, and he got five more problems under the same constraints. A total of four students per school were involved in the ciphering. The individual awards were based solely on the written test, while the team awards were based on the sum of the team's best four written test scores plus any ciphering points. This format served us well, and we decided to continue it for next year's Competition. Unfortunately, we had to wait two years for the next one!

The first Competition was held in January 2004, and we diligently worked towards the next one scheduled for January 2005. However, the day before the Competition was to take place, an ice storm came through the Atlanta area which forced the cancellation of all activities for the following day. No other date was available for us in the Spring of 2005, so we held all the materials until January 2006. (Which is why there is no 2005 Competition in this book!)

After two years of the same format, we were unhappy with the lack of team-oriented competition and we wanted to change it to encourage more team activity. We settled on an ARML-style competition. We introduced three divisions: middle school, junior varsity, and varsity (the middle school division is self explanatory, I think; the JV division is for competitors who have *not* started the study of precalculus, and the Varsity division is for anyone). We also introduced four rounds, three of which are team-oriented, designed for teams of six students from a school.

The Power Round In which a team must justify or prove their solutions to a series of related problems in a thirty-five minute time limit (40 points).

The Team Round In which a team must simply write answers to eight unrelated problems in a twenty minute time limit (5 points per correct answer).

The Relay Rounds In which the team splits into two groups of three where the first person must answer a problem, send that answer back to the second person where the second person must use the first person's answer to solve the second problem, then the second person passes that answer back to the third person who solves the final problem using the second person's answer. This is to be done within five minutes (5 points per correct final answer within three minutes and 2 points per correct final answer by five minutes – a total of 20 points).

The Individual Round In which each comeptitor is given two problems to answer in a seven-minute time limit, and then does this four times for a total of eight problems (one point per correct answer for a total of 48 points). The 8 problems increase in difficulty, with number 8 being the most difficult.

The individual awards were based solely on the individual round. With only eight problems in the individual round, we avoided numerous tie-breakers by ranking students not only on how many of these problems they answered correctly, but also on *which* problems they answered correctly. For example, if Pat got the first 6 out of the 8 correct, and Chris got the last 6 of the 8 correct, then Chris is ranked higher than Pat since Chris was able to answer the more difficult problems. Even though many tie-breakers were avoided with this system, some still had to be broken; this necessitated a tie-breaker round before the awards ceremony could begin. (The tiebreaker questions are not included in this book.) The team awards were based on the sum of all points from all rounds.

The new format in January 2007 was a success, and we endeavored to continue in January 2008 with the next Competition. Unfortunately, Mother Nature again dealt us a cold blow in the form of another ice storm! Once more, we were forced to cancel. The 2008 Competition was finally held in November 2008. Having learned our lesson, we scheduled the next Competition for December 2009.

This takes us to the present, and this book.

The problems represent the hard work of many students over the years. I edited the problems and wrote the 54 Relay Round problems; however, the authors of the other 700 problems in this book are the students listed below.

2004 Kenny Baskett, Max Bernardy, Mitch Costley, Amin Makhani, Ben Wu

2006 Max Bernardy, Mitch Costley, Kevin Todd, Ben Wu, Jodie Wu, Drew Yaun

2007 Sam Brotherton, Nayoon Kim, Amy Lanchester

2008 Sam Brotherton, Nayoon Kim, Jacob Kovac, Steven Rouk

2009 Kevin Brawley, Kathryn Daniel, Ashley Jackson, Jonathan Johnson, Kathryn Johnson, Patti Murphy, James Rives

Finally, I am so happy that I got to work with these mathematically gifted young people. They are truly amazing individuals. This book is dedicated to them.

(By the way, solutions to the 2004 and 2006 competitions are *not* included in this book just to give you a extra challenge!)

Chuck Garner
CONYERS, GA
APRIL 2010

Contents

A ANSWERS 323

4

The 2008 Rockdale Mathematics Competition

Last year's change in format did not decrease the participants, but changing from January to November did. The fourth RMC on November 1, 2008 was attended by 382 students from 32 different schools. Schools attending the RMC for the first time were Dodgen Middle School, Druid Hills High School, Fort Payne High School, Lakeside High School (from Atlanta), Pizitz Middle School, South Forsyth High School, and Vestavia Hills High School.

The top five individual winners of the tournament are listed in the table below.

	Middle School	Junior Varsity	Varsity
First place	Gil Goldshlager	Suyoung Jang	Eric Morphis
	Dickerson MS	*Vestavia Hills HS*	*Woodward Academy*
Second place	Tony Zeng	Owen Scott	Allen Park
	Pizitz MS	*Vestavia Hills HS*	*Walton HS*
Third place	Sean Eikhoff	Kyle Julian	Benjamin Hu
	Dickerson MS	*Vestavia Hills HS*	*Northview HS*
Fourth place	Botong Ma	Luke Bishop	Henry Mei
	Pizitz MS	*Vestavia Hills HS*	*Walton HS*
Fifth place	Jimmy Liu	Grant Carlson	Billy Dorminy
	Piztiz MS	*Westminster*	*Soli Deo Gloria*

The top two Middle School winners were the only middle school students to answer all eight Individual Round problems correctly. The top four JV winners answered 8, 7, 6, and 6 of the JV Individual Round problems correctly (and were the only JV students to correctly answer more than 5). The item difficulty for this year's Individual Round is in the table below.

Percentage of correct responses									
	#1	#2	#3	#4	#5	#6	#7	#8	Average
Middle School	62	36	38	83	23	41	11	4	38
Junior Varsity	33	47	8	19	25	47	12	11	25
Varsity	75	47	39	12	17	11	26	16	30

Awards were given to the top schools in each classification in each division. The overall top five middle school, junior varsity, and varsity teams are listed below. Schools were allowed to register more than one 6-student team.

	Middle School	Junior Varsity	Varsity
First place	Dickerson MS	Vestavia Hills HS A-team	Walton HS
Second place	Pizitz MS	Vestavia Hills HS C-team	Northview HS
Third place	Chamblee MS	Vestavia Hills HS B-team	Vestavia Hills HS
Fourth place	Hull MS	Walton HS	Chamblee HS
Fifth place	Dodgen MS	Chamblee HS	Lassiter HS

The Middle School tournament was a run-away for Dickerson, but the next four places were separated by only 9 points! All of the top five Middle School teams listed above obtained perfect scores on both the Power and Team Rounds (as well as Kendrick Middle School's team who got a perfect Power Round score). So the deciding factors were the Individual Round and the Relays. Another two problems correct, and the results would have been very different!

The JV tournament was the opposite of the Middle School: there were no perfect scores on the Power or Team Rounds.

Notable scores for the Varsity teams were the perfect Power Round scores obtained by Brookwood, Chamblee, Columbus, Lassiter, Vestavia Hills, Walton, and Woodward Academy, and the perfect Team Round scores obtained by Lassiter, Northview, and Woodward Academy. You may be wondering, if a team got perfect Power and Team scores, why that team may not be in the top five. Usually, when a school has perfect Power and Team and is not one of the top five, that team probably has one or two individuals who are veteran champions of mathematics competitions, while the rest of the team may mot be. This tournament is structured to reward good teams, not individuals. It is possible for an individual to answer the Team and Power alone, but the other teammates must be able to contribute to the Individual Round and the Relays. Without everyone working together, it is not possible for one or two students to "carry" a team to victory.

Average Team Scores					
	Individual	Team	Relay	Power	Total
Maximum Possible	48	40	20	40	148
All Middle School Teams	16.3	25.0	4.1	33.4	78.9
Top 5 Middle School Teams	30.4	40.0	11.4	40.0	121.8
All JV Teams	9.9	12.9	4.2	7.4	34.4
Top 5 JV Teams	19.6	22.0	9.8	9.6	61.0
All Varsity Teams	13.0	19.4	2.0	20.3	54.3
Top 5 Varsity Teams	29.0	36.0	8.2	39.8	112.6

2008 Middle School Power Round – The Study of Sets

In mathematics, a **set** is any collection of objects. For example, a set could include the numbers 1, 3, and 6; the letters f, d, q, and t; or even the shapes ◇ and ♡.

We may write down specifically the members of set (called **elements**) by enclosing the objects in curly braces. For instance, to define a set called S which contains the numbers 2, 4, and 6, we write $S = \{2, 4, 6\}$.

As you learn more and more about mathematics, many of the new concepts you learn can explained by relationships between sets. Next, we define some of those relationships.

- The **union** of two sets A and B, written $A \cup B$, is defined as the set containing all elements that are in either A or B (or both). For example, $\{1, 2, 3\} \cup \{2, 5, 7\} = \{1, 2, 3, 5, 7\}$.

- The **intersection** of two sets A and B, written $A \cap B$, is defined as the set containing all elements that are in both A and B. For example, $\{x, y, 3, 4, \} \cap \{a, b, c, 4, 5\} = \{4\}$.

- The **difference** between sets A and B, written $A - B$, is defined as the set containing all elements of A which are *not* in B. For example, $\{1, 2, 3, 4, 5, 6\} - \{1, 3, 5\} = \{2, 4, 6\}$.

You are to answer the following questions concerning sets. Show your work, and write clearly!

1. If $X = \{1, 10, 100, 1000\}$ and $Y = \{1, 5, 10, 15, 20\}$, then write the contents of the set $X \cup Y$.

2. If the set A contains all seven days of the week and B contains the five weekdays, then find $A - B$ and $A \cap B$.

3. Find a set V so that $V \cup \{3, 4, 5\}$ is the same as $V \cap \{3, 4, 5\}$.

4. Set E has six elements and set F has eight elements. E and F have exactly two elements in common. How many elements are in $E \cup F$?

5. Let S contain all integers greater than or equal to 1, and let T contain all the even integers greater than or equal to 2. If x is an element of the set $S - T$, what is the remainder when x is divided by two?

6. A set S is defined as a **subset** of a set A if $S \cap A = S$. If each subset must have at least one element, how many sets are subsets of $\{r, c, h, s\}$?

2008 Middle School Team Round

1. What is the smallest number of colors that can be used to color the figure so that no two adjacent regions have the same color? (*Note: Adjacent regions are those that share an edge.*)
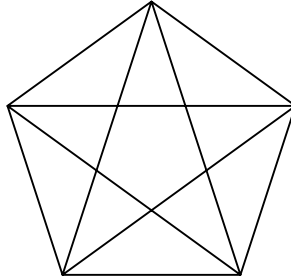
2. The area of a square is 17. What is the length of its diagonal?

3. Simplify to a single radical: $\left(\dfrac{\sqrt{54}}{\sqrt{27}}\right)^3 - \sqrt{72} + \sqrt{18}$.

4. What is the measure of the angle formed by the hands of a clock at 7:40?

5. Daniel has a bag containing x marbles. The marbles are colored blue, green, and yellow. He has a $\frac{2}{7}$ chance of taking a blue marble out of the bag, a $\frac{3}{5}$ chance of taking a green marble, and a $\frac{4}{35}$ chance of taking a yellow marble. What is the least possible value for x?

6. Given that $BD = 6$, $DC = 8$, and $AC = 15$, compute the measure of AE.
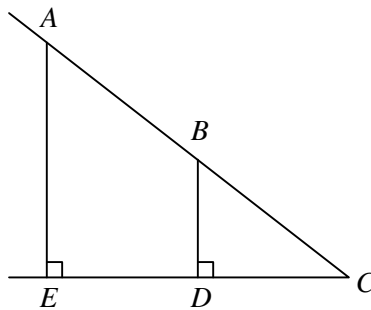
7. How many integers satisfy the inequality $|2x - 3| \le 4$?

8. Raymond owns a farm. On this farm there are chicken and cows. Raymond counts the heads and legs of all the chickens and cows and finds there are 22 legs and 7 heads. How many cows does Raymond have on his farm?