And Now, Everything Changes

The History of Mathematics, Part 11

Chuck Garner, Ph.D.

Department of Mathematics
Rockdale Magnet School for Science and Technology

March 1, 2021

Outline

Outline

Golden Age of Greece

- Approximately 600 BC to 400 AD
- Greece culture absorbed by Romans then Muslims

However, in the years 1 AD to 400 AD, there were some influential Greek mathematicians...

Heron

Heron of Alexandria

10 AD-75 AD

"The methods of dealing with these solids are, in view of their surprising character, referred to Archimedes by certain writers who give the traditional account of their origin. But whether they belong to Archimedes or another, it is necessary to give a sketch of these results as well."

Heron's Books

- *On the dioptra* (surveying and distances)
- *The pneumatica* (mechanical devices worked by air)
- *The automaton theatre*
- *Belopoeica* (engines of war)
- *Mechanica* (architecture)
- *Metrica*
- *Geometria*
- *Stereometrica*
- *Catoptrica* (mirrors and light)

Heron's Formula

From Book I of *Metrica*: if A is the area of a triangle with sides a, b, and c and $s = (a + b + c)/2$, then

$$A^2 = s(s - a)(s - b)(s - c).$$

Diophantus

Diophantus

200 AD-284 AD

- Not much is known about his life
- Lived, worked in Alexandria
- Gave “logistic” solutions in his work
- Was the first the solve problems in this manner
- Some consider him “father of algebra”
- Wrote the *Arithmetica*

The *Arithmetica*

DIOPHANTI
ALEXANDRINI
ARITHMETICORVM
LIBRI SEX.
ET DE NVMERIS MVLTANGVLIS
LIBER VNVS.

Nunc primùm Græcè & Latinè editi, atque absolutissimis Commentariis illustrati.

AVCTORE CLAVDIO GASPARE BACHETO
MEZIRIACO SEBVSIANO, V.C.

LVTETIAE PARISIORVM,
Sumptibus SEBASTIANI CRAMOISY, via
Iacobæa, sub Ciconiis.
M. DC. XXI.
CVM PRIVILEGIO REGIS.

First Latin edition (1621)

The *Arithmetica*

- Contains solutions to 130 problems in 13 books
- No general methods; methods are to be inferred from specific examples
- Solves
  - linear and quadratic equations
  - a special cubic equation
  - linear and quadratic indeterminate equations
- Only positive rational answers considered

The *Arithmetica*

Problem I-1 Find numbers such that the sum is 100 and the difference is 40.

Problem I-5 Find numbers such that the sum is 100 and the sum of the third and the fifth is 30.

Problem I-28 Find numbers such that the sum is 20 and the sum of the squares is 208.

The *Arithmetica*

Problem I-1 Find numbers such that the sum is 100 and the difference is 40.

Problem I-5 Find numbers such that the sum is 100 and the sum of the third and the fifth is 30.

Problem I-28 Find numbers such that the sum is 20 and the sum of the squares is 208.

Solution: Let numbers be x and y. Set $x = 10 + z$ and $y = 10 - z$. Then $(10 + z)^2 + (10 - z)^2 = 208$, or $200 + 2z^2 = 208$. Then $z = 2$ so $z = 12$ and $y = 8$.

Diophantus

Diophantus used the following abbreviations in the *Arithmetica*

the unknown	ς	sigma variant
unknown squared	Δ^{Υ}	*dunamis* ($\Delta\Upsilon$NAMIΣ), *power*
unknown cubed	K^{Υ}	*kubos* (KΥBOΣ), *cube*
fourth power	$\Delta^{\Upsilon}\Delta$	literally, *square-square*
fifth power	ΔK^{Υ}	*square-cube*
sixth power	$K^{\Upsilon}K$	*cube-cube*
minus	$\overset{\circ}{\Lambda}$	*leipis* (ΛΕΙΨΙΣ), *lacking*
the constant	$\overset{\circ}{\mathrm{M}}$	*monades* (MONAΔΕΣ), *units*

Diophantus

$x^3 + 13x^2 + 8x$ would be found in Diophantus as

$$\kappa^{\upsilon}\alpha\Delta^{\upsilon}\iota\gamma\varsigma\eta.$$

Diophantus

$x^3 + 13x^2 + 8x$ would be found in Diophantus as

$$K^{\Upsilon}\alpha\Delta^{\Upsilon}\iota\gamma\varsigma\eta.$$

$x^3 - 8x^2 + 2x - 3$ would be found in Diophantus as

$$K^{\Upsilon}\alpha\varsigma\beta\backslash\!\!\backslash\Delta^{\Upsilon}\eta\ \overset{\circ}{\mathrm{M}}\ \gamma.$$

- First instance of algebra, but was it really?
- Took a long time to catch on
- Diophantus was (apparently) unique in ancient Greece

The Last Ones

- Pappus (290-350), “last of the great Greek geometers”
- Theon (335-405), last librarian at Alexandria
- Proclus (411-485), last head of Plato’s Academy, extensive commentator

Hypatia

Hypatia of Alexandria

370-415

"Fables should be taught as fables, myths as myths, and miracles as poetic fantasies. To teach superstitions as truths is a most terrible thing. The child mind accepts and believes them, and only through great pain and perhaps tragedy can he be in after years relieved of them."

Reasons for Decline

- Fewer intellects
- All had been discovered already; commentating tradition began around 300 AD
- Romans
- Christians
- Arabs

Outline

The Romans and the Church

- Romans took what they needed to run an Empire
- Ignored the rest
- Romans did not encourage creative mathematics
- Christians destroyed secular learning
- The Church became the most powerful entity in Europe
- Only institutions of learning at this time were monasteries in Ireland

Anicus Boethius

Anicus Boethius

480-525

"The science of numbers ought to be preferred as an acquisition before all others, because of its necessity and because of the great secrets and other mysteries which there are in the properties of numbers. All sciences partake of it, and it has need of none."

Bede the Venerable

Bede the Venerable

673-735

"I have devoted my energies to the study of the scriptures, observing monastic discipline, and singing the daily services in church; study, teaching, and writing have always been my delight."

The Romans and the Church

- In 800, Charlemange became emporer of HRE
- Decreed that education is vital and math should be part of that
- Trivium (grammar, logic, rhetoric) and Quadrivium (astronomy, music, arithemtic, geometry)

Alcuin of York

Alcuin of York

735-804

"And those people should not be listened to who keep saying the voice of the people is the voice of God, since the riotousness of the crowd is always very close to madness."

Muslim Mathematics

- Muslim mathematicians translated many Greek works
- Spanish Moors helped Latin scholars translate much
- Slowly Europe regained the lost knowledge of the Greeks

Outline

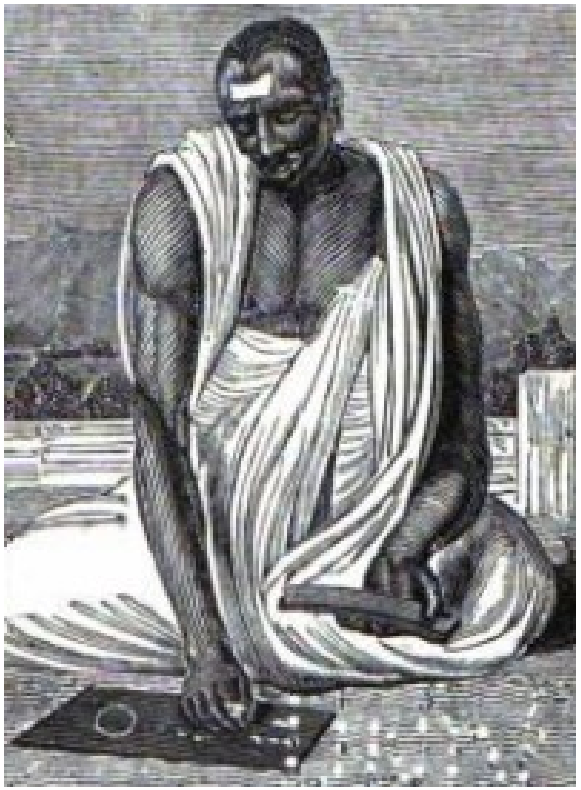
Āryabhaṭa

Aryabhata

476-550

"By the grace of Brahma, the sunken jewel of the best of true knowledge has been brought up by me out of the ocean of true and false knowledge by means of the boat of my intellect."

Brahmagupta



Brahmagupta

598-670

"As the sun eclipses the stars by its brilliancy, so the man of knowledge will eclipse the fame of others in assemblies of the people if he proposes algebraic problems, and still more if he solves them."

Brahmagupta

- Addition indicated by juxtaposition, subtraction by a dot over the subtrahend
- Multiplication by "*bha*" written after factors (from *bhavita*, product)
- Division by writing divisor beneath dividend
- Square root by "*ka*" before number (from *karana*, irrational)
- Unknown by "*yā*" (from *yāvattāvat*, so much as)
- Knowns prefixed by "*rū*" (from *rūpa*, absolute number)
- Additional unknowns by color; a second unknown might be "*kā*" (from *kālaka*, black)

Brahmagupta

- Addition indicated by juxtaposition, subtraction by a dot over the subtrahend
- Multiplication by "*bha*" written after factors (from *bhavita*, product)
- Division by writing divisor beneath dividend
- Square root by "*ka*" before number (from *karana*, irrational)
- Unknown by "*yā*" (from *yāvattāvat*, so much as)
- Knowns prefixed by "*rū*" (from *rūpa*, absolute number)
- Additional unknowns by color; a second unknown might be "*kā*" (from *kālaka*, black)

$8xy + \sqrt{10} - 7$ may appear as *yā kā* 8 *bha ka* 10 *rū* $\dot{7}$.

Outline

Ancient China

- Han Dynasty centralized rule of China
- Instituted civil service exams
- Spurred motivation to educate the populace
- Dynasties:
  - Qin, 221 BC-206 BC
  - Han, 206 BC-200 AD
  - Three Kingdoms, 220-265
  - Chin/North/South, 420-588
  - Sui, 581-618
  - Tang, 618-900
  - Song (North and South), 960-1279
  - Ming, 1368-1644

Liu Hui

Liu Hui

220-280

"Let us leave the problem to whoever can tell the truth."

Liu Hui

- First notable Chinese mathematician
- Wrote a commentary in 263 on *Jiuzhang suanshu* (*Nine Chapters on the Mathematical Art*)
- Dominated Chinese mathematical thought until 1600
  - Obtained $\pi \approx 3.141024$ from inscribing a 196-gon in a circle
  - Obtained area of a circle as "product of half the diameter and half the circumference"
- Also wrote *Haidao suanjing* (*Sea Island Mathematical Manual*)

A Calendar Problem

- Calendar problems were impetus for development of modular arithmetic and linear congruences
- 60-day cycle: *Shang yuan*–time when 60, winter solstice & new moon begin.
- Problem: "In a certain year, the winter solstice occurred r days into the cycle and s days after the new moon, then that year was N days after the *Shang yuan*, where N is a number such that 60 divides $(aN - r)$ and b divides $(aN - s)$, where a is the number of days in the year and b is the number of days between new moons."

Wang Xiaotong

- *Wang Xiaotong*, 580-640
- Wrote *Jigu suanjing* (*Continuation of Ancient Mathematics*)
- First instance of a solution to a cubic:
  Let a right angled triangle have sides a, b, and c where c is the hypotenuse. If a times b is $706 + \frac{1}{50}$, and if c is $36 + \frac{9}{10}$ more than a, then what are the values of the three sides?

Solution results in the equation

$$a^3 + \frac{a^2(c-a)}{2} = \frac{(ab)^2}{2(c-a)}$$

Qin Jiushao

Qin Jiushao

1202-1261

"At the time of the troubles with the barbarians, I spent several years on the remote frontier, without care for my safety among the arrows and stone missiles, I endured danger and unhappiness for ten years."

Qin Jiushao

- Terrible person, brilliant mathematician
- Called “China’s Archimedes”
- Wrote *Shushu jiuzhang* (*Mathematical Treatise in Nine Sections*) in 1247
- Solves quartic equations, systems of linear congruences, uses small circle for zero

Shushu jiuzhang

1. Linear congruences
2. Calendar
3. Surveying
4. Indirect measurement
5. Taxes
6. Money and grain
7. Fortifications and buildings
8. Military affairs
9. Commercial affairs

Shushu jiuzhang

Taxes are collected in seven cities, labeled A through G. Each city is owed the same amount, *N*, of taxes. Taxes are supposed to be paid in "full strings" of 100 coins, but money is scarce, so each city actually collects a smaller amount per string. Each city collects differently: 12 per 100 on A decreasing by 1 to 6 per 100 in G. All taxes are counted, and after full strings are counted, there are remainders: A has 10 left, D and G each have 4 left, E has 6 left, the others have none left. Find *N*.

Shushu jiuzhang

Taxes are collected in seven cities, labeled A through G. Each city is owed the same amount, *N*, of taxes. Taxes are supposed to be paid in "full strings" of 100 coins, but money is scarce, so each city actually collects a smaller amount per string. Each city collects differently: 12 per 100 on A decreasing by 1 to 6 per 100 in G. All taxes are counted, and after full strings are counted, there are remainders: A has 10 left, D and G each have 4 left, E has 6 left, the others have none left. Find *N*.

Answer: 26950

Cultural Influences

- Why did Chinese mathematics develop in the way it did?
- What prevented its development further?
- Was there transmission of ideas between Babylon, Greece, India, and China?

Outline

Homework

- Last-Minute Problems, #5 – due March 8
- A general survey of Middle Ages mathematics; *Math Through the Ages*, pages 25–32

Next: The House of Wisdom