The Heavens

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Ancient Greek Astronomy

Claudius Ptolemy and the Almagest

Ptolemy's Theorem

Homework

A Look Toward the Heavens The History of Mathematics, Part 10

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Outline

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Aristotle

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Aristarchus



Aristarchus of Samos 310 BC-230 BC

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Aristarchus

 Reasoned mathematically that Mercury and Venus revolved around the sun
 So the sun is much bigger than earth...
 So the earth revolves about the sun as well

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Aristarchus

- Reasoned mathematically that Mercury and Venus revolved around the sun
 So the sun is much bigger than earth...
 So the earth revolves about the sun as well
- First to give a heliocentric model of the solar system
- Archimedes uses Aristarchus' theory in The Sand-Reckoner
- Idea was not popular among the Greeks
- Used the equivalent of

$$\frac{\sin \alpha}{\sin \beta} < \frac{\alpha}{\beta} < \frac{\tan \alpha}{\tan \beta}$$

in his works (O $<\beta<\alpha<\pi/$ 2)

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Hipparchus



Hipparchus of Rhodes 190 BC-120 BC "Never deceive a friend."

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Hipparchus

- Careful and precise astronomical observer
- Determined the mean lunar month to within one second
- Accurate calculation of the inclination of the ecliptic
- Discovered procession of equinoxes
- Computed lunar parallax

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Hipparchus

- Careful and precise astronomical observer
- Determined the mean lunar month to within one second
- Accurate calculation of the inclination of the ecliptic
- Discovered procession of equinoxes
- Computed lunar parallax
- Catalogued 850 stars
- Advocated use of latitude and longitude
- May have been the first to introduce dividing a circle into 360 parts
- Constructed a table of chords

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Ptolemy



Claudius Ptolemy

90 AD-170 AD

"When I trace at my pleasure the windings to and fro of the heavenly bodies, I no longer touch the earth with my feet: I stand in the presence of Zeus himself and take my fill of ambrosia, food of the gods."

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Ptolemy

- Improved Hipparchus' table of chords
- Wrote the definitive Greek work on astronomy in 13 books
 - Mathematiki Syntaxis; trans. Mathematical Collection
 - Became known as Megisti Syntaxis; trans. Greatest Collection
 - Islamic mathematicians traslated it to al-Magisti
 - Latin translation of Arabic became Almagest

Retrogade Motion

retrograde motion East g b a West

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Epicycles

Deferent Earth Planet Epicycle

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Ptolemy

- Was not concerned with why only in a geometric model of motion
- Ptolemy calculated where planets and stars would be; to do this, he needed a way to measure chords in circles...
- Almagest surpassed all other astronomical works
- Did for astronomy what Elements did for geometry

The table gives the lengths of chords of all central angles of a circle from $\frac{1}{2}^{\circ}$ to 180° in half-degree intervals

- Radius of the circle is divided into 60 equal parts
- Chord lengths expressed in base-60 in terms of a radius-part
- ► Let's use "crd \alpha" to mean the length of a chord of angle \alpha

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Ptolemy's Theorem



So Ptolemy has

$$crd \, 36^\circ = 37^p \, 4' \, 55''$$

which means the chord of angle 36° is equal to

- 37 parts of the 60-part radius, plus
- 4/60 of one of these radius parts, plus
- ▶ 55/60 of one of the 60 parts of the radius part
- Note similarity to degrees, minutes, seconds

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- ► 55/60 of one of the 60 parts of the radius part
- Note similarity to degrees, minutes, seconds
- Let's relate chords to something more modern...

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| $\sin \alpha =$ | AM | 2 · AM | AB | $\operatorname{crd}(2\alpha)$ |
|-----------------|------|-------------------|------------|-------------------------------|
| | OA = | = <u>2 · OA</u> = | = <u> </u> | 120 |



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| cin a | AM | 2 · AM | AB | $\operatorname{crd}(2\alpha)$ |
|-----------------|------|-------------------|---------------|-------------------------------|
| $\sin \alpha =$ | OA = | = <u>2 · OA</u> = | = <u> =</u> = | 120 |

So the Table of Chords gives sines in quarter-degree intervals from 0° to 90°.

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Ptolemy's Theorem

This geometrical gem is not found in the Elements

Theorem

In a cyclic quadrilateral the product of the diagonals is equal to the sum of the products of the two pairs of opposite sides.

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Proof of Ptolemy's Theorem



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Let *E* be on \overline{BD} such that $\angle BCE = \angle ACD$. Then $\triangle BCE \sim \triangle ACD$, so that

Proof of Ptolemy's Theorem



AD/AC = BE/BC, whence $AD \cdot BC = BE \cdot AC$. From similar triangles $\triangle ABC$ and $\triangle DEC$, we have AB/AC = DE/DC, whence $AB \cdot DC = DE \cdot AC$.

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Proof of Ptolemy's Theorem



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AD/AC = BE/BC, whence $AD \cdot BC = BE \cdot AC$. From similar triangles $\triangle ABC$ and $\triangle DEC$, we have AB/AC = DE/DC, whence $AB \cdot DC = DE \cdot AC$. Therefore

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 $AD \cdot BC + AB \cdot DC$ = BE \cdot AC + DE \cdot AC = AC(BE + DE) = AC \cdot BD.

Ptolemy's Corollaries

Corollary (1)

If a and b are the chords of two arcs of a circle of unit radius, then

$$s = \frac{a}{2}\sqrt{4-b^2} + \frac{b}{2}\sqrt{4-a^2}$$

is the chord of the sum of the two arcs.

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Ptolemy's Corollaries

Corollary (2)

If $a \ge b$ are the chords of two arcs of a circle of unit radius, then

$$d=\frac{a}{2}\sqrt{4-b^2}-\frac{b}{2}\sqrt{4-a^2}$$

is the chord of the difference of the two arcs.

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Corollary (3)

If t is the chord of a minor arc of a circle of unit radius, then

$$\mathsf{h}=\sqrt{\mathsf{2}-\sqrt{\mathsf{4}-\mathsf{t}^2}}$$

is the chord of half the arc.

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Calculating the Chord of 1 Degree

- From "Incommensurables" an isosceles triangle with equal sides 1 and vertex angle 36° has base equal to the golden ratio 0.6180
- Hence, in a circle of unit radius,

 $\operatorname{crd} 36^{\circ} = 60 \times 0.6180 = 37.082.$

In a circle of unit radius, crd 60° = 1, so by Corollary
 2:

$$\operatorname{crd} 24^{\circ} = \operatorname{crd} (60^{\circ} - 36^{\circ}) = 22.918.$$

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 2:

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m 22.918}.$$

Using Corollary 3, we may successively find chords of 12°, 6°, 3°, $\frac{3}{2}°$, and $\frac{3}{4}°$, obtaining

$$\operatorname{crd} \frac{3}{2}^{\circ} = 1.5708$$
 and $\operatorname{crd} \frac{3}{4}^{\circ} = 0.7854$.

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Ptolemy's Theorem

Calculating the Chord of 1 Degree By the relation $\frac{\operatorname{crd} \alpha}{\operatorname{crd} \beta} < \frac{\alpha}{\beta}$, for $o^{\circ} < \beta < \alpha < 90^{\circ}$, we have

$$\frac{\operatorname{crd} 1^{\circ}}{\operatorname{crd} (3/4)^{\circ}} < \frac{1}{3/4} = \frac{4}{3},$$

or

$$\operatorname{crd} 1^{\circ} < rac{4}{3} \operatorname{crd} \left(rac{3}{4}
ight)^{\circ} = rac{4}{3} imes 0.7854 = 1.0472.$$

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Ptolemy's Theorem

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$$\frac{\operatorname{crd} 1^{\circ}}{\operatorname{crd} (3/4)^{\circ}} < \frac{1}{3/4} = \frac{4}{3},$$

or

$$\operatorname{crd} 1^{\circ} < \frac{4}{3} \operatorname{crd} \left(\frac{3}{4} \right)^{\circ} = \frac{4}{3} \times 0.7854 = 1.0472.$$

Also,

$$\frac{\text{crd}\,(3/2)^{\circ}}{\text{crd}\,1^{\circ}} < \frac{3/2}{1} = \frac{3}{2},$$

or

$$\operatorname{crd} 1^{\circ} > \frac{2}{3} \operatorname{crd} \left(\frac{3}{2}\right)^{\circ} = \frac{2}{3} \times 1.5708 = 1.0472.$$

Hence. crd $1^{\circ} = 1.0472$.

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Ninth century copy, in Greek, from the Vatican Library – oldest known copy of the *Almagest*

Ptolemy's Table of Chords

| Ptolemy's Table of Chords | | | | | | | | | |
|---------------------------|--------|--------------------|---------|---------------------|------|-------------------------|----|------|---|
| Arc (°) | ¢ | Chord ₆ | 0 | Chord ₁₀ | | Sixtieths ₆₀ | | | Sixtieths ₁₀ |
| 0.5 | 0 | 31 | 25 | 0.523611 | 0 | 1 | 2 | 50 | 0.017226 |
| 1.0 | 1 | 2 | 50 | 1.047222 | 0 | 1 | 2 | 50 | 0.017226 |
| 1.5 | 1 | 34 | 15 | 1.570833 | 0 | 1 | 2 | 50 | 0.017226 |
| 2.0 | 2 | 5 | 40 | 2.094444 | 0 | 1 | 2 | 50 | 0.017226 |
| 2.5 | 2 | 37 | 4 | 2.617778 | 0 | 1 | 2 | 48 | 0.017226 |
| 3.0 | 3 | 8 | 28 | 3.141111 | 0 | 1 | 2 | 48 | 0.017226 |
| 3.5 | 3 | 39 | 52 | 3.664444 | 0 | 1 | 2 | 48 | 0.017226 |
| 4.0 | 4 | 11 | 16 | 4.187778 | 0 | 1 | 2 | 47 | 0.017226 |
| 4.5 | 4 | 42 | 40 | 4.711111 | 0 | 1 | 2 | 47 | 0.017226 |
| 5.0 | 5 | 14 | 4 | 5.234444 | 0 | 1 | 2 | 46 | 0.017226 |
| 5.5 | 5 | 45 | 27 | 5.757500 | 0 | 1 | 2 | 45 | 0.017226 |
| 6.0 | 6 | 16 | 49 | 6.280278 | 0 | 1 | 2 | 44 | 0.017226 |
| 6.5 | 6 | 48 | 11 | 6.803056 | 0 | 1 | 2 | 43 | 0.017226 |
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A general survey of Greek mathematics; Math Through the Ages, pages 15-24

Next: And Now, Everything Changes

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